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The George Washington University
Institute for Management Science and Engineering
Program in Logistics

ERRATA FOR

OPTIMAL TREATMENT LEVELS OF A
STREAM POLLUTION ABATEMENT SYSTEM UNDER
THREE ENVIRONMENTAL CONTROL POLICIES
PART I: SOLUTION AND ANALYSIS OF CONVEX
EQUIVALENTS OF ECKER'S GP MODELS USING SUMT

Technical Paper Serial T-387

19 January 1979

by

Anthony V. Fiacco
Abolfazi Ghaemi

Page Number	Correction
iii	The page number for Figure 3 should be 8 (not 6).
7	(i) In Equation (2) and in the second line following this equation, change e to $\underline{E}$.
	(ii) In the definition of L_b, change "at the bottom of" to "before entering."
9	In the second line preceding and the first line following Equation (5), the small s_i should be a capital S_i.
10	Replace the last sentence by "The variable t_{ij}, the fraction of BOD remaining in the process in Reach i after Process j, is a positive fraction, positivity being enforced by the model formulation; thus,"
11	(i) Replace line one by: "$0 < t_{ij} \leq 1$ for all i, j."
	(ii) In the inequality beginning with u_{21} (line 9), replace m_2, appearing over the first product sign, by m_1.

Page Number	Correction

12

(i) In the third equation following Equation (1), the subscript r of R should be changed to w.

(ii) In Equation (2), change e to $\underline{E}$.

(iii) Replace the parenthetical remark at the bottom of the page by: "For notational simplicity the subscript i indicating the reach number is suppressed, except for V_{R_0}, L_{b_0}, and D_{b_0}, where the subscript zero indicates specified levels of the respective quantities just prior to entering Reach 1."

13

Replace this page by the attached Page 13.

14

Delete the parameters L_{b_0} and D_{b_0} and their definitions.

18

In Constraint (20), replace the subscript 2 by 6, throughout.

21

In the first line replace the coefficient 1.68 by 16.8.

35

(i) In the (fourth) equation for C_{31}, replace the subscript capital S_1 of K by small s_1, and the subscript r_1 of R by w_1.

(ii) In the (fifth) equation for L_1, replace e_1 by $\underline{E}_1$.

(iii) In the (sixth) equation for e_1, replace e_1 by $\underline{E}_1$ and the subscript i of t by 1.

(iv) In the last two inequalities on the page, replace the subscript i of t by 1.

(v) In the last inequality on Page 35, the quantity multiplying C_{21} should be D_{b_0} rather than b_0.

36

(i) In the first line, replace the subscript i of t by 1.

(ii) In the (fourth) equation for C_{32}, replace the subscript capital S_2 of K by small s_2, and the subscript r_2 of R by w_2.

Page Number	Correction

36   (iii) In the (fifth) equation for L_2, replace e_2 by $\underline{E}_2$.

     (iv) In the (sixth) equation for e_2, replace e_2 by $\underline{E}_2$.

     (v) In the (last) equation for L_1, replace the subscript i of t by 1.

37   In the first equation, first replace the small c_{11} by capital C_{11}. Then, under the product sign, replace j-1 by j=1 and, following the product sign, replace the subscript i of t by 1.

Parameters representing physical measurements involved in the dissolved oxygen deficit Equation (1):

V_{R_0} – Flow rate of the river before entering Reach 1, in 10^6 gallons/day

L_{b_0} – Initial BOD level of the stream before entering Reach 1, in mg/ℓ

D_{b_0} – Initial oxygen deficit of the stream before entering Reach 1, in mg/ℓ

K – Deoxygenation constant, in day^{-1}

r – Reaeration constant, in day^{-1}

t – Flow time along the reach, in days

F – Fraction of river bottom covered with sludge

Q – A coefficient in the sludge term, determined empirically in day^{-1}

K_s – Oxygen uptake rate per unit area of stream bottom surface, in gm/m^2/day

R_w – Hydraulic radius of the river cross section, in meters

V_E – Volume of the effluent released into the river, in 10^6 gallons/day

E – BOD concentration of effluent, before treatment, in mg/ℓ

Aside from the above parameters which are involved in the oxygen sag equation of each reach, the following parameters will also enter into the formulation of the model at various stages.

Parameters representing quantities determined by management decisions or feasibility considerations:

S – Maximum allowable oxygen deficit, in mg/ℓ

$\underline{P}$ – Minimum required or feasible BOD removal by a specified sequence of treatment components in a given reach, a fraction

$\overline{P}$ – Maximum possible or feasible BOD removal by a specified sequence of treatment components in a given reach, a fraction

$\underline{PP}$ – Minimum required or feasible BOD removal by a specified single treatment component in a given reach, a fraction

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Technical paper

56p.

Serial-T-387
19 January 1979

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

Program in Logistics

Abstract
of
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The existing approaches to the mathematical modeling and optimization
of a water pollution problem are briefly surveyed. A proposed geometric
programming model of a water pollution and treatment system, which easily
lends itself to the theory and application of nonlinear programming sensi-
tivity analysis techniques, is studied in detail. As in previous work by
the author of the model, the optimal waste treatment facilities along the
Upper Hudson River are presented for three different environmental policies,
leaving the sensitivity analysis study for a following report. Previous
results were obtained using a geometric programming code, while the present
study makes use of the SUMT code. It is shown that the policy of fixed
dissolved oxygen requirement yields the minimum annual waste treatment cost,
relative to two other policies. Moreover, it is shown that a variable
dissolved oxygen policy yields relatively uniform treatment levels in the
treatment plants at appreciably reduced cost compared to the costs involved
in strict uniform treatment policy. The results of the present study are
consistent with, but not identical to, the findings reported in previous
work. The discrepancy in the coefficients involved in the dissolved oxygen
constraints used by these two studies is the main reason for the differ-
ences observed. A listing of a computer program developed to calculate
these coefficients is included.

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GEOMETRIC PROGRAMMING	DISSOLVED OXYGEN REQUIREMENT
CONVEX PROGRAMMING	BIOCHEMICAL OXYGEN DEMAND
SUMT	OPTIMAL WASTE TREATMENT
WATER POLLUTION ABATEMENT SYSTEM	SYSTEM DESIGN

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The existing approaches to the mathematical modeling and optimization of a water pollution problem are briefly surveyed. A proposed geometric programming model of a water pollution and treatment system, which easily lends itself to the theory and application of nonlinear programming sensitivity analysis techniques, is studied in detail. As in previous work by the author of the model, the optimal waste treatment facilities along the Upper Hudson River are presented for three different environmental policies,

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20. Abstract - continued

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a computer program developed to calculate these coefficients is
included.

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1. Introduction

The present paper reports on the solution and analysis of a geometric programming water pollution control model [1] formulated by J. G. Ecker and applied by him to a problem involving data for the Upper Hudson River. A basic premise motivating the environmental pollution problem is that the pollutant level should not "significantly" diminish the "quality of life." Stated more simply and affirmatively in the context of an important criterion of stream water pollution, the dissolved oxygen of the stream should be kept above a certain level. The standard for an acceptable level of dissolved oxygen in a stream is usually set by the various environmental protection agencies. It is important to note that, other than dissolved oxygen (DO), there are many other possible measures of water quality, e.g., temperature, taste, color, odor, dissolved chloride, or turbidity. However, the dissolved oxygen appears to be the most common water quality measure. The reason for this, as Sobel [2] points out, may be paraphrased as follows:

Dissolved oxygen fluctuates in response to many of the
phenomena reflected in the other measures; and it may serve

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as a surrogate for them. DO improvement programs often have side benefits associated with the improvements in all the measures mentioned. The DO content of the water plays an important role in supporting aquatic life. Its control may be accomplished in a variety of ways that have motivated rather sophisticated problem formulations.

A major cause of the violation of dissolved oxygen standards in streams is the release of industrial wastes. Those wastes usually contain organic materials which, in the course of decomposition, consume and diminish the dissolved oxygen content of the water and endanger aquatic life and the quality of the surrounding environment.

A standard measure for organic waste quality is the level of Biochemical Oxygen Demand (BOD) concentration, defined as the quantity of dissolved oxygen in the stream required to stabilize the organic material in the discharge over a five-day period at $20°$ c.

In order to meet a specified dissolved oxygen level, the discharged waste volume and its BOD concentration must obviously be limited. Fortunately, the organic wastes released into the streams do not consume oxygen indefinitely. This is due to the fact that these materials eventually decompose and stabilize. Moreover, due to reaeration along the stream, the consumed oxygen is constantly replenished by absorption of oxygen from the air.

Using the dissolved oxygen content as a pollution criterion, numerous attempts have been made to model and control the quality of a stream, using mathematical programming. Such attempts have generally strived for one or both of the following goals:

i) Provide guidelines for allocation of treatment facilities at each discharge point.

ii) Determine the optimal waste treatment level at each discharge point.

These goals have to be met such that a certain dissolved oxygen standard is achieved with a minimal expenditure of resources.

It is clear that the DO profile equation along a stream and the waste treatment cost function play important roles in any water quality management model. The nonlinearity of the dissolved oxygen profile (oxygen sag)

Equation [3] and treatment cost function [4], [5], [6] render such models
inherently nonlinear. However, in the early attempts by Deininger [7] and
Deininger and Sobel [8], the problem was cast as a linear programming model.
Deininger structured linear programming models which utilize various approxi-
mations of the differential equation used to describe the dissolved oxygen
profile of the streams. Following such pioneering efforts, the problem was
further investigated in the format of a linear programming model by Sobel [2],
Kerri [9], Revelle, Loucks, and Lynn [10], [11], Graves and Hatfield [12],
Arbabi and Elzinga [13] and others. As McNamara [14] pointed out, however,
linear programming formulations unduly restrict the form of the functional
relationships involved in the model.

A large number of nonlinear models were subsequently developed to
optimize the abundant variations of the problems that arise in water quality
management. Liebman and Lynn [15], Shih and Krishnan [16], Keegan and Leeds
[17], Meier and Beightler [18], [19] and Dysart [20] mainly applied dynamic
programming to model the problem.

Ecker and McNamara [21], Ecker [1] and McNamara [14] were able to
formulate an important class of pollution control and facility design problems
as geometric programming models. In [21] they use the data provided in the
dynamic programming approach of Shih and Krishnan [16] and obtain essentially
the same optimal design. Among many appealing features of the Ecker models
are that they readily lend themselves to the sensitivity analysis study
approach motivated by the work of Fiacco and McCormick [22] and developed by
Fiacco [23]. Ecker [1] points out that "the model is particularly useful
for sensitivity analysis involving changes in the stream standards."

The present paper reports on the application of the above model for
the optimal facility design and treatment level allocations along the Upper
Hudson River, for three alternative pollution control policies. As noted,
these problems have been solved by the author of the models using a geometric
programming code, while the present study makes use of a general nonlinear
programming formulation and code, SUMT [24]. A computer routine was developed
to calculate the coefficients involved in the oxygen deficit constraints
and the resulting data manipulations have yielded somewhat different (and
apparently more accurate) coefficients than those used in the previous study.

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The results of the present study show that for the Upper Hudson River the optimal annual pollution control cost is minimum under the fixed dissolved oxygen requirement policy and maximum under the uniform treatment requirement policy. The third policy, which allows a variable dissolved oxygen requirement, yields the same stream dissolved oxygen concentration as the uniform treatment policy, but under considerably reduced annual treatment costs. Moreover, this policy yields relatively uniform treatment levels that are easier to administer than the other policies. These results are consistent with, but not identical to, the findings reported in [1].

2. Model Description

2.1 Background. As mentioned earlier, in order to meet the dissolved oxygen standard set by environmental rules, the discharged waste volume and its BOD concentration into the streams must be limited. To maintain a tolerable release of BOD concentration, an industry must first process wastes in treatment plants. Figure 1 depicts a typical arrangement of the facilities in a treatment plant. As noted, the segment of the stream from one treatment plant to the next is called a "reach." (More generally, a reach is taken to be that portion of a stream along which there is not a sudden and significant change in the water quality and stream parameters.) This allows one to divide the stream under study into various segments (reaches) and use fixed parameters to define the oxygen profile along each segment. As indicated in Figure 1, the initial treatment process is called a "primary treatment," intermediate processes are called "secondary treatments" and the last process is called a "final" or "advanced treatment" process. Primary and secondary treatments taken together are, albeit somewhat ambiguously, referred to as "total secondary treatments."

Depending on the BOD discharge, length, and other characteristics of the reach, the minimum oxygen deficit may either occur at the end of each reach or within the reach, as shown in Figures 2a and 2b, respectively. In the formulation of the model under study it is assumed that the DO profile takes the form given in Figure 2a.

To meet environmental standards, the dissolved oxygen along the stream must at all times be above the minimum allowable dissolved oxygen level dictated. In terms of oxygen deficit, the oxygen deficit at any
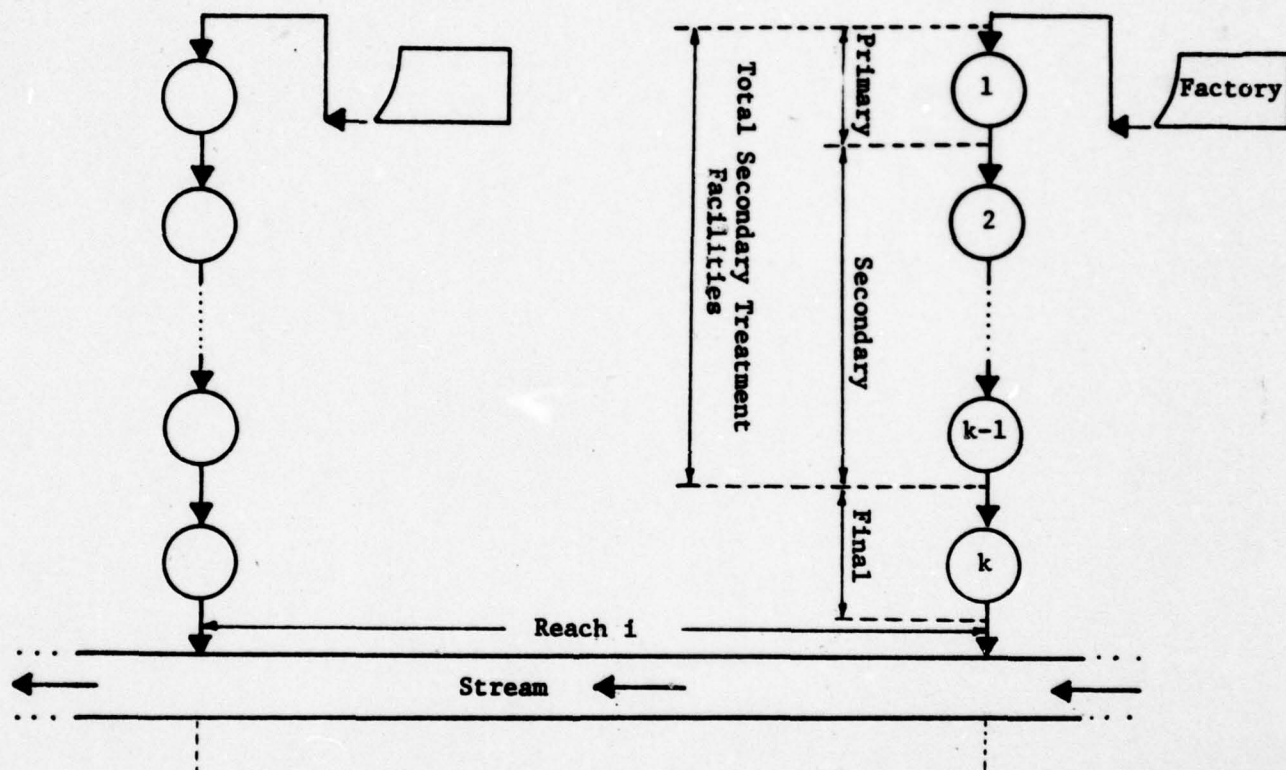
Figure 1. A Typical Waste Treatment Component Arrangement.
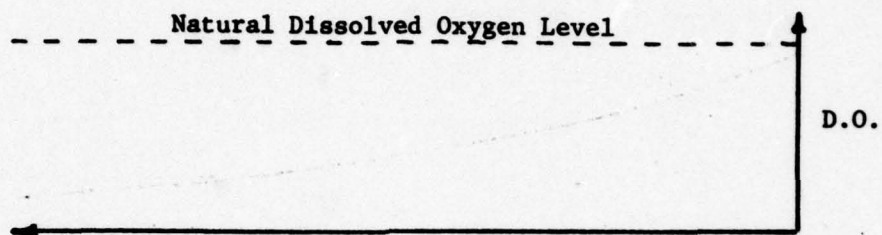
Figure 2a. Dissolved Oxygen Profile Along Reach i.
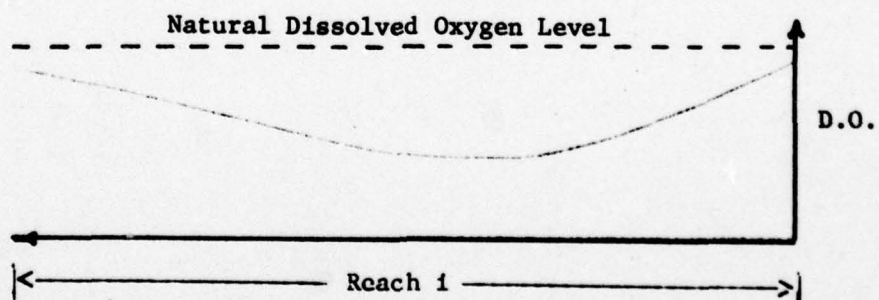
Figure 2b. Dissolved Oxygen Profile Along Reach i.
(An Alternative Profile)

time and at any point along the stream should not exceed the maximum allowable oxygen deficit dictated by the environmental standards.

The equation for the dissolved oxygen deficit D_t at time t along a particular reach is

$$D_t = c_1 L + c_2 D + c_3 \tag{1}$$

where

L = BOD concentration of the stream at the top of the reach (after waste release)

D = Dissolved oxygen deficit at the top of the reach, and c_1, c_2 and c_3 are coefficients which depend upon time and reach parameters to be discussed in the next section.

The BOD concentration L of the stream at the top of each reach can be calculated from the following simple mass balance equation

$$L = \frac{V_E}{V_E + V_R} e + \frac{V_R}{V_E + V_R} L_b \tag{2}$$

where

V_E = Volume of effluent per day into the stream

e = BOD concentration of effluent into the stream

V_R = Volume of stream per day before entering the reach

L_b = BOD concentration of stream at the bottom of the reach

The BOD concentration L_{b_i} at the bottom of Reach i can be calculated if the BOD concentration L_i at the top of Reach i is known, as follows

$$L_{b_i} = c_4 L_i \tag{3}$$

where c_4 is a coefficient which depends upon time and reach parameter.

Hence, for a sequence of reaches, if we know the BOD concentration and oxygen deficit at the top of the first reach, we can readily obtain the oxygen deficit profile along the entire stream (for all reaches), provided

that reach parameters, stream volume, effluent volume, and waste discharged by each treatment plant are known.

2.2 The Model. The model under study is a geometric programming model developed for allocating treatment requirements along the stream so as to meet dissolved oxygen standards for each reach, while minimizing the total annual cost of all treatment activities.

The problem variable t_{ij} is defined as the fraction of BOD remaining in the effluent after it passes through the jth treatment process of Reach 1. Figure 3 is a schematic depiction of the treatment facilities for Reach 1 of the Upper Hudson River.
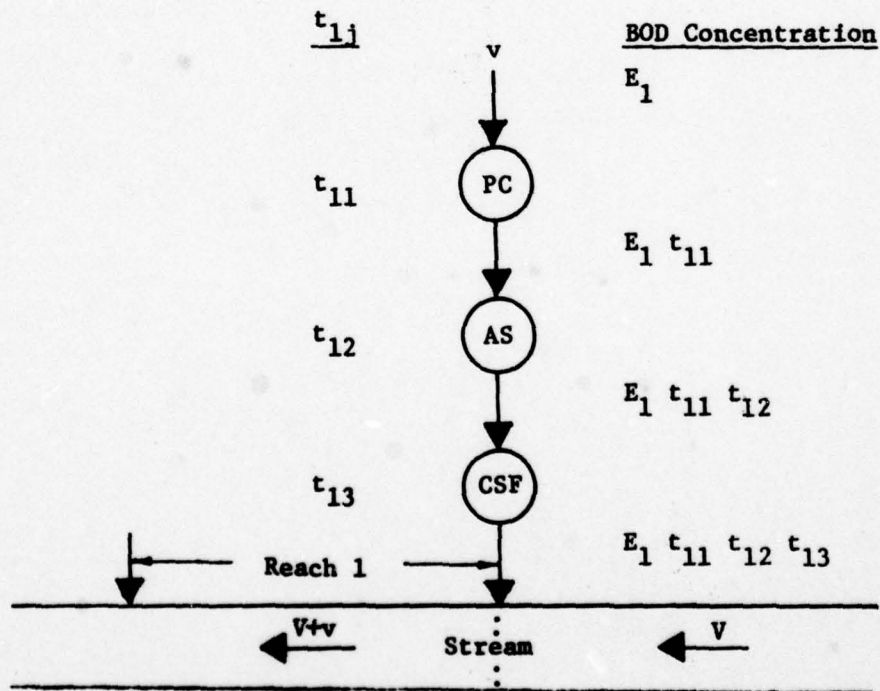
Figure 3. Configuration of Design Treatment Components for Reach 1.
(See page 14 for abbreviations)

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E_1 is the BOD concentration of the waste of volume v entering the waste treatment system 1, so $t_{11}E_1$ is the BOD concentration of the treated waste remaining after process 1 and entering process 2, $t_{11}t_{12}E_1$ leaves process 2 and enters process 3 and $t_{11}t_{12}t_{13}E_1$ leaves process 3 and enters the stream. As mentioned before in the formulation of the model, it is assumed that the maximum oxygen deficit within each reach will occur at the end of the reach.

It is stipulated that the yearly operating cost $\bar{c}_{ij}$ of process j in Reach i depends upon t_{ij} as follows

$$\bar{c}_{ij} = c_{ij}t_{ij}^{-\alpha_{ij}}$$

where c_{ij} and α_{ij} are nonnegative constants that can be determined once the kind of treatment facility is specified. If there are n reaches and m_i processes in reach i then the total yearly operating cost F to be minimized is

$$F = \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij}\, t_{ij}^{-\alpha_{ij}} \tag{4}$$

which constitutes the objective function of the model.

There are four types of constraints involved in the formulation of the model:

a. Dissolved oxygen deficit constraint.

Dissolved oxygen deficit along Reach i must be less than maximum allowable oxygen deficit s_i along Reach i. This constraint, after appropriate manipulation, is

$$u_{i1} \prod_{j=1}^{m_1} t_{1j} + u_{i2} \prod_{j=1}^{m_2} t_{2j} + \cdots + u_{ii} \prod_{j=1}^{m_i} t_{ij} \leq 1 \tag{5}$$

where the u_{ij}'s are positive constants depending on s_i and the specific parameters of Reach i and those

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upstream of Reach i. For illustrative purposes, the
dissolved oxygen deficit constraints of Problem P_1 for
Reaches 1 and 2 and the calculation of the u_{ij} for
these reaches are given in Appendix 1.

b. Combined treatment requirement.

The fraction of BOD removed by a sequence of treatment
components in Reach i may be bounded from above or below
by specified fractions $\bar{P}_i$ or $\underline{P}_i$ respectively, yielding
constraints of the form

$$1 - \prod_{j \in K_i^*} t_{ij} \leq \bar{P}_i$$

or $\qquad\qquad\qquad$ (6)

$$1 - \prod_{j \in K_i^*} t_{ij} \geq \underline{P}_i$$

where K_i^* is the set of indices of the subject treatment
components in Reach i.

c. Operating range constraint for components of
    treatment facilities in Reach i.

The fraction of BOD removed by component j in Reach i
may be bounded from above or below by specified fractions
$\overline{PP}_{ij}$ or $\underline{PP}_{ij}$, respectively, yielding constraints of the
form

$$1 - t_{ij} \leq \overline{PP}_{ij}$$

or

$$1 - t_{ij} \geq \underline{PP}_{ij}$$

d. Natural constraints.

The variable t_{ij} , the fraction of BOD remaining in the
process in Reach i after process j , is a nonnegative fraction,
thus

$$0 \leq t_{ij} \leq 1 \qquad \text{for all } i,j .$$

After some simple algebraic manipulations, the given model for a stream involving n reaches, with Reach i containing m_i processing components, can be formulated as

$$\text{minimize} \quad F = \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij} t_{ij}^{-\alpha_{ij}}$$

subject to

    a. Dissolved oxygen deficit constraints (5)

$$u_{11} \prod_{j=1}^{m_1} t_{1j} \leq 1$$

$$u_{21} \prod_{j=1}^{m_2} t_{1j} + u_{22} \prod_{j=1}^{m_2} t_{2j} \leq 1$$

$$\begin{matrix} \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{matrix}$$

$$u_{n1} \prod_{j=1}^{m_1} t_{1j} + u_{n2} \prod_{j=1}^{m_2} t_{2j} + \cdots + u_{nn} \prod_{j=1}^{m_n} t_{nj} \leq 1$$

    b. Constraints on possible combinations of processes (6)

$$(1-\bar{P}_i) \prod_{j \in K_i^*} t_{ij}^{-1} \leq 1$$

$$(1-\underline{P}_i)^{-1} \prod_{j \in K_i^*} t_{ij} \leq 1 \quad , \text{ for all } i$$

    c. Constraints on possible operating ranges (7)

$$(1-\overline{PP}_{ij}) t_{ij}^{-1} \leq 1$$

$$(1-\underline{PP}_{ij})^{-1} t_{ij} \leq 1 \quad \text{ for desired } i \text{ and } j$$

d. Natural constraints (8)

$$0 < t_{ij} \leq 1 \qquad \text{for all } i \text{ and } j$$

2.3 Model Parameters. As indicated, the model describes a multistage activity, the stages or "reaches" essentially being contiguous sections of the stream. The activity begins at a given point along the stream at a given time, with organic wastes being first treated and subsequently deposited into the stream. The next deposit and treatment occurs at some point down stream that defines the end of the given stage and beginning of the next.

The parameters defining the model are mainly those involved in the oxygen profile (sag) equation. For a given reach at a given time t the dissolved oxygen deficit D_t , given in the previous section, is

$$D_t = c_1 L + c_2 D + c_3 \tag{1}$$

where

$$c_1 = K(r-K)^{-1}(e^{-Kt} - e^{-rt})$$

$$c_2 = e^{-rt}$$

$$c_3 = QFK_s (1-e^{-rt})/R_r \quad \text{(sludge term)}$$

and the BOD concentration L of the stream at the top of the reach, as indicated in the last section, is

$$L = \frac{V_E}{V_E + V_R} e + \frac{V_R}{V_E + V_R} L_b \tag{2}$$

and BOD concentration L_{b_i} at the bottom of Reach i is

$$L_{b_i} = c_4 L_i \tag{3}$$

where $c_4 = e^{-k_i t_i}$. The parameters involved in those equations are defined as follows. (For notational simplicity the subscript i indicating the reach number is suppressed.)

- 12 -

K   - Deoxygenation constant, in day^{-1}

r   - Reaeration constant, in day^{-1}

t   - Flow time along the reach, in days

F   - Fraction of river bottom covered with sludge

Q   - A coefficient in the sludge term, determined empirically in day^{-1}

K_S - Oxygen uptake rate per unit area of stream bottom surface, in $gm/m^2/day$

R_r - Hydraulic radius of the river cross section, in meters

V_E - Volume of the effluent released into the river, in 10^6 gallons/day

E   - Concentration of effluent, before treatment, in mg/ℓ

e   - Concentration of effluent, after treatment, in mg/ℓ

V_R - Flow rate of the river before entering the reach, in 10^6 gallons/day

Aside from the above parameters which are involved in the oxygen sag equation of each reach the following parameters will also enter into the formulation of the model at various stages.

S   - Maximum allowable oxygen deficit, in mg/ℓ

$\underline{P}$ - Minimum required or feasible BOD removal by a specified sequence of treatment components in a given reach, a fraction

$\bar{P}$ - Maximum possible or feasible BOD removal by a specified sequence of treatment components in a given reach, a fraction

$\underline{PP}$ - Minimum required or feasible BOD removal by a specified single treatment component in a given reach, a fraction

$\overline{PP}$ - Maximum possible or feasible BOD removal by a specified single treatment component in a given reach, a fraction

L_{b_0} - Initial BOD level of the stream before entering Reach 1, in mg/ℓ

D_{b_0} - Initial oxygen deficit of the stream before entering Reach 1, in mg/ℓ

3. Formulation of the Models for the Upper Hudson River

The pictorial depiction of the design components for treatment facilities and corresponding annual operating cost data considered by the author of the model along the Upper Hudson River is given in Figure 4. A description of the treatment components, shown in the figure from left to right is as follows.

Reach 1 - Primary Clarifier (PC), Activated Sludge (AS), Coagulation/Sedimentation/Filteration (CSF)

Reach 2 - PC, Trickling Filter (TF), Activated Lagoon (AL), CSF

Reach 3 - PC, TF, AS, Carbon Absorption (CA)

Reach 4 - PC, AL, CSF

Reach 5 - PC, TF, AS, CSF

Reach 6 - PC, TF, AL, CSF

As in reference [1], under three different treatment policies, three problems, P_1, P_2, and P_3 are respectively formulated which involve the optimization of the total yearly operating cost of the above treatment system.

The first policy, a fixed dissolved oxygen requirement policy, requires that a minimum stream dissolved oxygen concentration level of 6.2 mg/ℓ be maintained along the entire stream. The second policy, a uniform BOD treatment policy, requires that each treatment plant, (i.e., the collection of treatment components at a given discharge point) remove at least 95 percent of the BOD concentration of its effluent. The third policy relaxes the
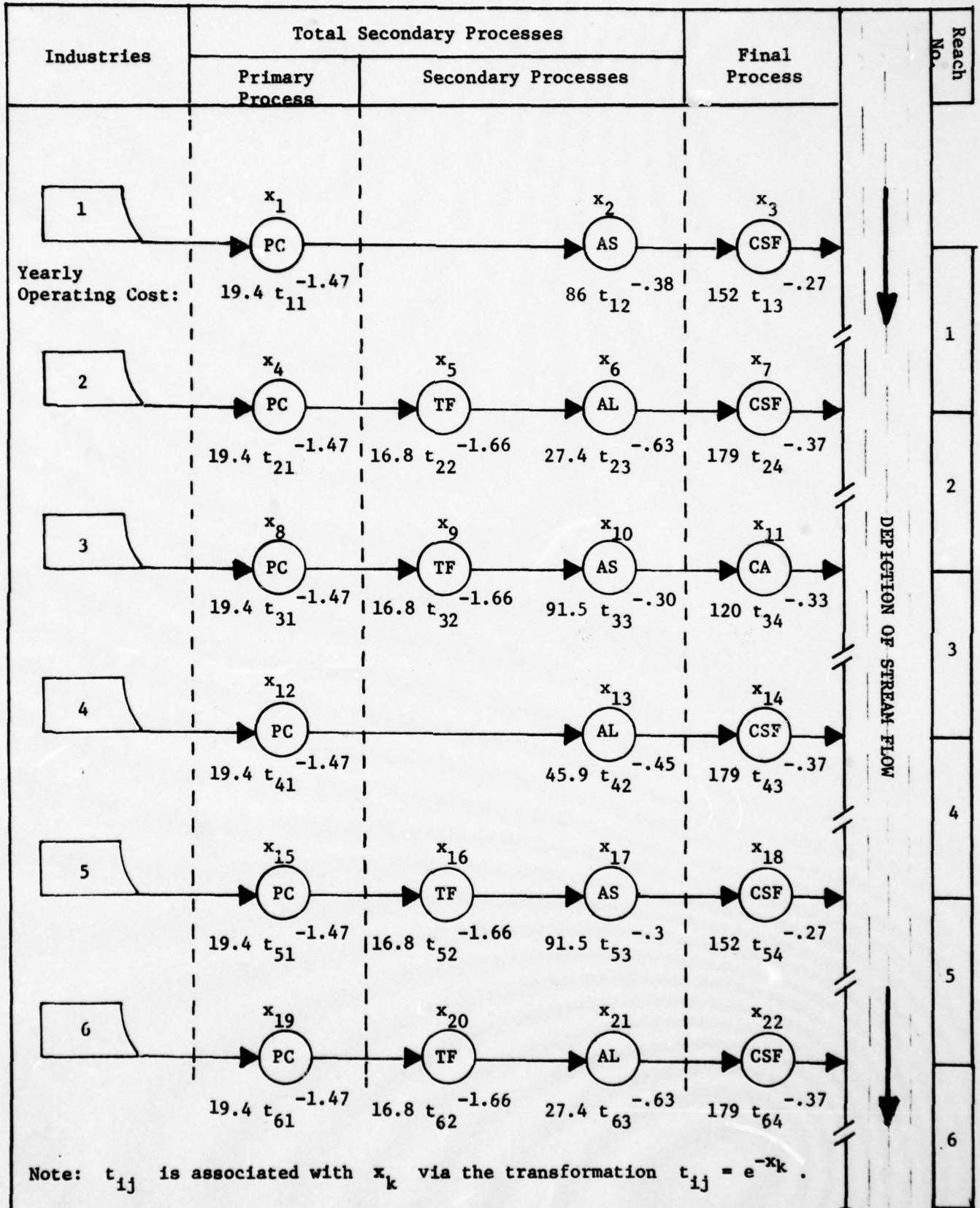
Figure 4. Configuration of the Design Treatment
Facilities Along the Upper Hudson River
(For abbreviations see page 14)

requirement of the first two policies, but uses the levels of dissolved
oxygen that resulted for a given reach under the second policy as the minimum
required dissolved oxygen standard for that reach. This allows changes in
both the oxygen deficit and treatment levels from reach to reach, while guaran-
teeing (with the data as given) that the oxygen standard required by the first
policy is more than satisfied. The input data for the above problems were
extracted from Reference [1] and supplemented in part by J. G. Ecker, the
author of the model. The formulations of Problems P_1, P_2, and P_3 follow.

Problem P_1: Corresponding to a "fixed dissolved oxygen
requirement policy" of 6.2 mg/ℓ along the
river.

(Dimensions: 22 variables, 42 constraints)

$$\text{Minimize } F = 19.4\, t_{11}^{-1.47} + 86\, t_{12}^{-.38} + 152\, t_{13}^{-.27} +$$

$$19.4\, t_{21}^{-1.47} + 16.8\, t_{22}^{-1.66} + 27.4\, t_{23}^{-.63} + 179\, t_{24}^{-.37} +$$

$$19.4\, t_{31}^{-1.47} + 16.8\, t_{32}^{-1.66} + 91.5\, t_{33}^{-.30} + 120\, t_{34}^{-.33} +$$

$$19.4\, t_{41}^{-1.47} + 45.9\, t_{42}^{-.45} + 179\, t_{43}^{-.37} +$$

$$19.4\, t_{51}^{-1.47} + 16.8\, t_{52}^{-1.66} + 91.5\, t_{53}^{-.3} + 152\, t_{54}^{-.27} +$$

$$19.4\, t_{61}^{-1.47} + 16.8\, t_{62}^{-1.66} + 27.4\, t_{63}^{-.63} + 179\, t_{64}^{-.37}$$

subject to:

a. Dissolved oxygen deficit constraint, i.e.,
dissolved oxygen deficit for all reaches ≤ 2.18 mg/ℓ .
(Type (a), Section 2.2)

- 16 -

1. $u_{11}t_{11}t_{12}t_{13} \leq 1$

2. u_{21} " $+u_{22}t_{21}t_{22}t_{23}t_{24} \leq 1$

3. u_{31} " $+u_{32}$ " $+u_{33}t_{31}t_{32}t_{33}t_{34} \leq 1$

4. u_{41} " $+u_{42}$ " $+u_{43}$ " $+u_{44}t_{41}t_{42}t_{43} \leq 1$

5. u_{51} " $+u_{52}$ " $+u_{53}$ " $+u_{54}$ " $+u_{55}t_{51}t_{52}t_{53}t_{54} \leq 1$

6. u_{61} " $+u_{62}$ " $+u_{63}$ " $+u_{64}$ " $+u_{65}$ " $+u_{66}t_{61}t_{62}t_{63}t_{64} \leq 1$

             b. Constraints on combination of processes
                 (Type(b), Section 2.2)

7. $.1\ t_{11}^{-1}\ t_{12}^{-1} \leq 1$ at most 90 percent removal by total secondary treatment on reach 1

8. $.15\ t_{21}^{-1}\ t_{22}^{-1}\ t_{23}^{-1} \leq 1$ at most 85 percent removal by total secondary treatment on reach 2

9. $.15\ t_{31}^{-1}\ t_{32}^{-1}\ t_{33}^{-1} \leq 1$ at most 85 percent removal by total secondary treatment on reach 3

10. $.15\ t_{41}^{-1}\ t_{42}^{-1} \leq 1$ at most 85 percent removal by total secondary treatment on reach 4

11. $.15\ t_{51}^{-1}\ t_{52}^{-1}\ t_{53}^{-1} \leq 1$ at most 85 percent removal by total secondary treatment on reach 5

12. $.15\ t_{61}^{-1}\ t_{62}^{-1}\ t_{63}^{-1} \leq 1$ at most 85 percent removal by total secondary treatment on reach 6

13. $1.4275\ t_{61}\ t_{62} \leq 1$ at least 30 percent removal by first two components in reach 6

             c. Operating range constraint for components
                 of treatment facilities (Type (c), Section 2.2)

14. $1.25\ t_{41} \leq 1$ at least 20 percent removal by first component in reach 4

d. Redundant constraints (added to prevent numerical overflow)

15. $e^{-80} \, t_{11}^{-1} \, t_{12}^{-1} \, t_{13}^{-1} \quad \leq 1$

16. $e^{-80} \, t_{21}^{-1} \, t_{22}^{-1} \, t_{23}^{-1} \, t_{24}^{-1} \leq 1$

17. $e^{-80} \, t_{31}^{-1} \, t_{32}^{-1} \, t_{33}^{-1} \, t_{34}^{-1} \leq 1$

18. $e^{-80} \, t_{41}^{-1} \, t_{42}^{-1} \, t_{43}^{-1} \quad \leq 1$

19. $e^{-80} \, t_{51}^{-1} \, t_{52}^{-1} \, t_{53}^{-1} \, t_{54}^{-1} \leq 1$

20. $e^{-80} \, t_{21}^{-1} \, t_{22}^{-1} \, t_{23}^{-1} \, t_{24}^{-1} \leq 1$

e. Natural constraints (Type (d), Section 2.2)

21 - 42. $\qquad 0 < t_{ij} \leq 1 \qquad i = 1, 6, \quad j = 1, m_i$

The coefficients u_{ij} in the oxygen deficit constraints (1-6) are complicated and lengthy functions of nearly all the problem parameters. To conduct a detailed sensitivity analysis (the subject of a follow-up report) with respect to the problem parameters it was necessary to delineate the functions defining these coefficients in terms of the problem parameters. Explicit derivation of these functions by hand proved extremely tedious and time consuming and evaluations were prone to computational errors. The task was virtually prohibitive for more than three reaches, since the equations for a given reach involve quantities that depend on results obtained in all previous reaches. To resolve this difficulty, the computer program listed in Appendix 2 was written to define and calculate the u_{ij} coefficients in terms of the problem parameters. The computed u_{ij} for Problem P_1, along with a listing of the parameter values are given in Appendix 2. Although the program was written to handle the problem involving six reaches and the

Upper Hudson River data base, it can readily be modified to handle other data and other likely variations that might appear in other models.

Problem P_2: Corresponding to a "uniform BOD treatment policy" of 95 percent removal for all reaches. (Dimensions: 22 variables, 48 constraints)

Minimize F   (See Problem P_1)

Subject to:

    a. Constraints 1-20 of Problem P_1

    b. At least 95 percent treatment in all reaches, i.e.,

21. $20\ t_{11}\ t_{12}\ t_{13} \le 1$

22. $20\ t_{21}\ t_{22}\ t_{23}\ t_{24} \le 1$

23. $20\ t_{31}\ t_{32}\ t_{33}\ t_{34} \le 1$

24. $20\ t_{41}\ t_{42}\ t_{43} \le 1$

25. $20\ t_{51}\ t_{52}\ t_{53}\ t_{54} \le 1$

26. $20\ t_{61}\ t_{62}\ t_{63}\ t_{64} \le 1$

    c. Nonnegativity constraints of Problem P_1

Based on the assumptions and results of this problem, the dissolved oxygen level (hence the oxygen deficit S_i) in each reach was calculated.

The oxygen deficit levels so obtained were used as the new oxygen deficit standards for Problem P_3 and new u_{ij} coefficients were calculated. Thus, Problem P_3 was formulated as follows.

Problem P_3:   Corresponding to the policy of using the levels
of dissolved oxygen that resulted for a given
reach for Problem P_2 as the minimum required
dissolved oxygen standard for that reach.
(Dimensions: 22 variables, 42 constraints)

Minimize F   (See Problem P_1)

Subject to:

   a.   Constraints 1-6 of Problem P_1 with coefficients

        u_{ij} calculated using new S_i

   b.   Constraints 7-20 of Problem P_1

   c.   Nonnegativity constraints.

The coefficients u_{ij} along with the required data for this calculation
are listed in Appendix 4.

By the usual transformation, i.e., letting

$$x_k = -\ln t_{ij} \quad (i=1,6, \quad j=1,m_i \quad \text{and} \quad k=1, \sum_{i=1}^{6} m_i) ,$$

as labelled in Figure 4, the geometric programming problems P_1 , P_2 , and
P_3 were converted to convex programming problems. The resulting equivalent
problems, labelled C_1 , C_2 , and C_3 , respectively, follow.

Problem C_1:   Convex Equivalents of Problem P_1.
(Dimensions: 22 variables, 42 constraints)

$$\text{Minimize } F = 19.4e^{1.47x_1} + 86e^{.38x_2} + 152e^{.27x_3}$$

$$+ 19.4e^{1.47x_4} + 16.8e^{1.66x_5} + 27.4e^{.63x_6} + 179e^{.37x_7}$$

$$+ 19.4e^{1.47x_8} + 1.68e^{1.66x_9} + 91.5e^{.30x_{10}} + 120e^{.33x_{11}}$$

$$+ 19.4e^{1.47x_{12}} + 45.9e^{.45x_{13}} + 179e^{.37x_{14}}$$

$$+ 19.4e^{1.47x_{15}} + 16.8e^{1.66x_{16}} + 91.5e^{.30x_{17}} + 152e^{.27x_{18}}$$

$$+ 19.4e^{1.47x_{19}} + 16.8e^{1.66x_{20}} + 27.4e^{.63x_{21}} + 179e^{.37x_{22}}$$

subject to:

        a. Dissolved oxygen deficit constraint, i.e., the dissolved oxygen deficit for all reaches ≤ 2.18 mg/ℓ . (Type (a), Section 2.2)

1. $-u_{11}e^{-(x_1+x_2+x_3)} + 1$ ≥ 0

2. $-u_{21}$ " $-u_{22}e^{-(x_4+x_5+x_6+x_7)} + 1$ ≥ 0

3. $-u_{31}$ " $-u_{32}$ " $-u_{33}e^{-(x_8+x_9+x_{10}+x_{11})} + 1$ ≥ 0

4. $-u_{41}$ " $-u_{42}$ " $-u_{43}$ " $-u_{44}e^{-(x_{12}+x_{13}+x_{14})} + 1$ ≥ 0

5. $-u_{51}$ " $-u_{52}$ " $-u_{53}$ " $-u_{54}$ " $-u_{55}e^{-(x_{15}+x_{16}+x_{17}+x_{18})} + 1$ ≥ 0

6. $-u_{61}$ " $-u_{62}$ " $-u_{63}$ " $-u_{64}$ " $-u_{65}$ " $-u_{66}e^{-(x_{19}+x_{20}+x_{21}+x_{22})}$ ≥ 0

        b. Constraints on combination of processes (Type (b), Section 2.2)

7. $-(x_1+x_2) + \ell n10 \geq 0$ (At most 90 percent removal by total secondary treatments in Reach 1.)

8. $-(x_4+x_5+x_6) + \ln \frac{1}{.15} \geq 0$

9. $-(x_8+x_9+x_{10}) + \ln \frac{1}{.15} \geq 0$

10. $-(x_{12}+x_{13}) + \ln \frac{1}{.15} \geq 0$

11. $-(x_{15}+x_{16}+x_{17}) + \ln \frac{1}{.15} \geq 0$

12. $-(x_{19}+x_{20}+x_{21}) + \ln \frac{1}{.15} \geq 0$

(At most 85 percent removal by total secondary treatments in Reaches 2 – 6.)

13. $x_{19} + x_{20} + \ln .7 \geq 0$ (At least 30 percent removal by first two components in Reach 6.)

c. Operating range constraint for components of treatment facilities (Type (c), Section 2.2)

14. $x_{12} + \ln .8 \geq 0$ (At least 20 percent removal by first component in Reach 4.)

d. Redundant constraints added to prevent numerical overflow.

15. $-(x_1+x_2+x_3) + 80 \geq 0$

16. $-(x_4+x_5+x_6+x_7) + 80 \geq 0$

17. $-(x_8+x_9+x_{10}+x_{11}) + 80 \geq 0$

18. $-(x_{12}+x_{13}+x_{14}) + 80 \geq 0$

19. $-(x_{15}+x_{16}+x_{17}+x_{18}) + 80 \geq 0$

20. $-(x_{19}+x_{20}+x_{21}+x_{22}) + 80 \geq 0$

e. Natural constraints (Type (d), Section 2.2)

$x_i \geq 0$ $i = 1,22$

Problem C_2:   Convex Equivalent of Problem P_2

           (Dimensions: 22 variables, 48 constraints)

Minimize F   (See Problem C_1)

subject to:

     a.   Constraints 1-20 of Problem C_1

     b.   At least 95 percent treatment in all reaches, i.e.,

21. $x_1 + x_2 + x_3 + \ln .05 \geq 0$

22. $x_4 + x_5 + x_6 + x_7 + \ln .05 \geq 0$

23. $x_8 + x_9 + x_{10} + x_{11} + \ln .05 \geq 0$

24. $x_{12} + x_{13} + x_{14} + \ln .05 \geq 0$

25. $x_{15} + x_{16} + x_{17} + x_{18} + \ln .05 > 0$

26. $x_{19} + x_{20} + x_{21} + x_{22} + \ln .05 \geq 0$

     c.   Nonnegativity constraints

Problem C_3:   Convex Equivalent of Problem P_3

           (Dimensions: 22 variables, 42 constraints)

Maximize F   (See Problem C_2)

subject to:

     a.   Constraints 1-6 of Problem C_1 with coefficients u_{ij} of Problem P_3

     b.   Constraints 7-20 of Problem C_1

     c.   Nonnegativity constraints

4. Optimal Solutions of the Upper
   Hudson River Models

The convex programming problems C_1, C_2, and C_3 developed in the
previous section were solved with SUMT for the optimum treatment levels
and the annual waste treatment cost. The approximate resulting optimal
solutions obtained for the respectively equivalent problems, P_1, P_2, and
P_3 are as follows.

4.1 Problem P_1

- **Optimal treatment levels** (using a fixed dissolved
  oxygen standard of 6.2 mg/ℓ along the entire stream)

  t^*_{ij}

	j=1	j=2	j=3	j=4
i=1	.7236	.4095	.6617	--
i=2	.7729	.7854	.2471	.4343
i=3	.8640	.8668	.4303	1.
i=4	.8000	.3229	1.	--
i=5	1.	1.	1.	1.
i=6	.7729	.7854	.2471	.9551

- Annual waste treatment cost = $1,837,851

- Binding constraints: (6,8,12 and 14)

The fact that a given component t_{ij} is equal to unity at the
optimal solution indicates that the corresponding treatment facility need
not be included (if this is operationally feasible) in the plant design,
since that component contributes nothing to the treatment process when it
is operated optimally. The calculated annual waste treatment cost should
then be reduced accordingly. In the above table, these components were
not removed in calculating the total costs. Suppose, however, that we
conclude that the last treatment facilities in Reaches 3 and 4 and all
facilities in Reach 5 are not needed. Making this adjustment, the optimal
waste treatment cost as shown in Table 1 is $1,253,140 per year.

Table 1

OPTIMAL PLANT BOD REMOVAL LEVELS
AND COSTS FOR PROBLEM P_1

(Using a fixed dissolved oxygen standard
of 6.2 mg/ℓ along the entire stream)

Reach No.	Total BOD Removal (%)	Cost $/Year	Redundant Components
1	80.39	321,870	None
2	93.48	363,230	None
3	67.77	163.190	CA
4	74.17	103,260	CSF
5	0.00	--	All Components
6	85.67	301,590	None

Total $1,253,140

If we exclude the last component in Reach 6, as is done in [1], the cost would further decrease to $1,253,140 − $182,060 = $1,071,070 per year. (The solution obtained in [1] was $1,106,000 per year.) However, based on our results we do not see the justification for the given exclusion.

As mentioned previously, we have traced back the discrepancy of our solution with that obtained in [1] to the differences in the values of the coefficients u_{ij} used to calculate the oxygen deficit constraints. Our calculated u_{ij}, along with the list of required input data, are included in the Appendix.

4.2 Problem P$_2$

- **Optimal treatment levels** (using 95% uniform treatment for all reaches)

  t^*_{ij}

	j=1	j=2	j=3	j=4
i=1	.6338	.2452	.3216	--
i=2	.7729	.7854	.2471	.3333
i=3	.7720	.7845	.2477	.3333
i=4	.7792	.2057	.3333	--
i=5	.7720	.7845	.2477	.3333
i=6	.7730	.7854	.2471	.3333

- Optimal annual waste treatment cost = $2,330,279

- Binding constraints: (8,9,10,11,12,21,22,23,24, 25,26)

Unlike the results obtained for Problem P$_1$, none of the components of the solution matrix are unity. Thus, we cannot exclude any treatment component with this policy. As shown in Table 2, if we exclude the treatment facilities of Reach 5, as is done in Reference [1], then the optimal annual waste treatment cost would reduce to $2,330,279 - $397,080 = $1,933,199, which is in close agreement with the result, $1,926,000 per year, given in Reference [1]. (In fact, if we use our Table 2 calculation, we get $2,322,580 - $397,080 = $1,925,900, which agrees almost exactly with [1].)

Table 2

OPTIMAL PLANT BOD REMOVAL LEVELS
AND COSTS FOR PROBLEM P_2

(Using a 95% uniform treatment
for each reach)

Reach No.	Total BOD Removal (%)	Cost $/Year	Redundant Components
1	95	391,120	None
2	95	388,300	"
3	95	365,030	"
4	95	393,150	"
5	95	397,080	"
6	95	388,300	"
	Total	$2,322,980*	

*This value is about $7,300 less than that found by direct solution. This discrepancy is attributed to the translation errors involved in hand calculation of the costs on the above table.

All of the uniform treatment constraints, i.e., constraints 21-26, are binding. On the other hand, the dissolved oxygen constraints 1-6 are more than satisfied. As in [1], this suggests using the dissolved oxygen levels resulting under the uniform treatment policy as the minimum standard for each reach, thus, allowing the levels of treatment to vary from reach to reach. Under such conditions one would hope that the optimal cost would be lower than that obtained under the policy of uniform treatment while yielding an optimal treatment schedule that would be nearly uniform. (Uniform policies are apparently easier to administer.) The following optimal solution of problem P_3 confirms these expectations.

4.3 Problem P_3

- **Optimal treatment levels** (using oxygen deficit standards that may vary from reach to reach)

  t_{ij}

	j=1	j=2	j=3	j=4
i=1	.6336	.2449	.3210	--
i=2	.7729	.7854	.2471	.1782
i=3	.7722	.7847	.2480	.7720
i=4	.7290	.2057	.9322	--
i=5	.9814	.9727	.8271	1.
i=6	.7729	.7854	.2471	.3920

- Annual waste treatment cost = $2,147,336

- Binding constraints (1,6,8,9,10,12,14)

As it is seen, the last component in reach 5 is redundant since $t_{5,4} = 1$. Thus the optimal waste treatment cost =

$2,147,336 - $152,000 = $1,995,336 per year. The solution obtained in Reference [1] was $1,460,000 per year. Here again, the discrepancy between our solution and that of Reference [1] is partly due to the differences in the values used as the coefficients u_{ij} of the dissolved oxygen constraints, and partly due to the fact that, for reasons explained in Section 4.1, we have not excluded the treatment facilities in Reach 5 in our calculations.

Table 3

OPTIMAL PLANT BOD REMOVAL LEVELS
AND COSTS FOR PROBLEM P_3

Reach No.	Total BOD Removal (%)	Cost $/Year	Redundant Components
1	95.02	391,300	None
2	97.33	458,380	"
3	88.40	323,210	"
4	86.02	308,090	"
5	21.04	134,390	CSF
6	94.12	372,650	None

Total $1,988,020*

*This value is about $7,316 less than that found by direct solution. This discrepancy is attributed to the truncation errors involved in hand calculation of the costs in the above table.

As it is seen in Table 3, the treatment levels for all reaches except Reach 5, which contributes a moderate level of pollutant to the stream, are nearly uniform. Also the optimal treatment cost is $1,995,336 per year compared to that for uniform treatment of $2,330,279 per year. Therefore, under the stream standards (dictated by the solution of P_2) we may achieve a relatively uniform treatment policy and at the same time save $2,330,279 - $1,995,336 = $334,943 per year.

A comparison of the above results shows that the minimum annual waste treatment cost is obtainable under the fixed dissolved oxygen policy of 6.2 mg/ℓ . The cost incurred under the 95 percent uniform treatment policy is by far the highest, although this policy is the easiest to monitor and administer. The same stream standards achieved under uniform treatment level policy can be met under an almost uniform treatment level policy, with appreciable savings in treatment cost. The annual saving under such a policy

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for the model under study is of the order of $335,000, amounting to a 14.4 percent savings in annual treatment cost.

5. Conclusions

Based on the previous results and discussions, the following conclusions are immediate.

- Among the three different policies studied for optimal water pollution control and treatment facility allocation, the policy requiring a fixed dissolved oxygen of 6.2 mg/ℓ along the stream yields the least annual waste treatment cost, $1,253,140.

- The requirement of a 95 percent uniform waste treatment level in all reaches, a policy easier to monitor and administer, requires an additional annual cost of $1,077,140 when this requirement is added to the fixed oxygen level standard.

- The same dissolved oxygen standards achieved under 95 percent uniform treatment level policy can be met under a near-uniform treatment level policy, with savings of about $335,000 per year, amounting to a 14.4 percent saving in the annual treatment cost obtained by a uniform treatment policy.

- The above findings are consistent with, but not identical to, the findings reported in [1]. The reason for discrepancies in the optimal solution is attributed to the discrepancy between u_{ij} , the coefficients appearing in the dissolved oxygen constraints, used by these two studies.

- The computer routine SUMT solved these geometric programming convex equivalent models without difficulty. The efficiency, however, compared to that of a special purpose GP code for solving the original problem, has yet to be determined. Apart from this, the given model can now be appreciably modified to include much more general functions (than those required by the G. P. model), since SUMT assumes no such special structure.

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Appendix 1

DISSOLVED OXYGEN DEFICIT CONSTRAINTS OF
REACHES 1 AND 2 FOR PROBLEM P_1

APPENDIX 1

Dissolved Oxygen Deficit Constraint for Reach 1 in Problem P_1 .

The oxygen deficit D_{b_1} at the bottom of Reach 1 is calculated from Equation (1) (Section 2.3) as follows

$$D_{b_1} = C_{11} L_1 + C_{21} D_{b_0} + C_{31}$$

where

$$C_{11} = K_1 (r_1 - K_1)^{-1} \left(e^{-K_1 t_1} - e^{-r_1 t_1} \right)$$

$$C_{21} = e^{-r_1 t_1}$$

$$C_{31} = Q_1 F_1 K_{S_1} \left(1 - e^{-r_1 t_1} \right) / R_{r_1}$$

$$L_1 = \frac{1}{V_{E_1} + V_{R_0}} \left(V_{E_1} e_1 + V_{R_0} L_{b_0} \right) \text{(using Equation 2, Section 2.3)}$$

and

$$e_1 = E_1 \sum_{j=1}^{3} t_{ij} .$$

Imposing the dissolved oxygen deficit constraint at the bottom of Reach 1, i.e., $D_{b_1} \leq S_1$, yields

$$C_{11} \frac{1}{V_{E_1} + V_{R_0}} \left(V_{E_1} E_1 \prod_{j=1}^{3} t_{ij} + V_{R_0} L_{b_0} \right) + C_{21} D_{b_0} + C_{31} \leq S_1 .$$

After simplification, we obtain

$$\left(C_{11} \frac{V_{E_1}}{V_{E_1} + V_{R_0}} E_1 \right) \left(S_1 - C_{21} b_0 - C_{31} - C_{11} \frac{V_{R_0} L_{b_0}}{V_{E_1} + V_{R_0}} \right)^{-1} \prod_{j=1}^{3} t_{ij} \leq 1 ,$$

using the data provided in Appendix 3. The coefficient of $\prod\limits_{j=1}^{3} t_{ij}$

in the above constraint (which is labelled u_{11} in the model) was

calculated to be 0.7756. Thus, the oxygen deficit constraint for Reach 1
becomes simply

$$.7756 \ t_{11} t_{12} t_{13} \leq 1$$

Dissolved Oxygen Deficit Constraint for Reach 2 in Problem P_1.

The oxygen deficit D_{b_2} at the bottom of Reach 2 is calculated
from Equation 1 (Section 2.3) as follows:

$$D_{b_2} = C_{12} \ L_2 + C_{22} \ D_{b_1} + C_{32}$$

where

$$C_{12} = K_2 (r_2 - K_2)^{-1} \left(e^{-K_2 t_2} - e^{-r_2 t_2} \right)$$

$$C_{22} = e^{-r_2 t_2}$$

$$C_{32} = Q_2 F_2 K_{S_2} \left(1 - e^{-r_2 t_2} \right) / R_{r_2}$$

$$L_2 = \frac{1}{V_{E_2} + V_{R_1}} \left(V_{E_2} e_2 + V_{R_1} L_{b_1} \right) \text{(using Equation 2, Section 2.3)}$$

and

$$e_2 = E_2 \prod_{j=1}^{4} t_{2j}$$

$$V_{R_1} = V_{R_0} + V_{E_1}$$

$$L_{b_1} = e^{-K_1 t_1} L_1 \quad \text{(using Equation 3, Section 2.3)}$$

$$L_1 = \frac{1}{V_{E_1} + V_{R_0}} \left(V_{E_1} E_1 \prod_{J=1}^{3} t_{ij} + V_{R_0} L_{b_0} \right)$$

with

$$D_{b_1} = c_{11} \frac{1}{V_{E_1} + V_{R_0}} \left(V_{E_1} E_1 \prod_{j-1}^{3} t_{ij} + V_{R_0} L_{b_0} \right) + C_{21} D_{b_0} + C_{31}$$

as shown for Reach 1.

Imposing the dissolved oxygen deficit constraint at the bottom of Reach 2, i.e., $D_{b_2} \leq S_2$, and using the data provided in Appendix 3, the above constraint reduces to

$$u_{21} \prod_{j=1}^{3} t_{1j} + u_{22} \prod_{j=1}^{4} t_{2j} \leq 1 \, ,$$

where u_{21} and u_{22} are calculated to be .8544 and .9172, respectively. Thus, the oxygen deficit constraint for Reach 2 becomes

$$.8544 \; t_{11} t_{12} t_{13} + .9172 \; t_{21} t_{22} t_{23} t_{24} \leq 1 \, .$$

Appendix 2

PROGRAM LISTING TO CALCULATE u_{ij}

```
$JOB
      REAL K3(6),AO
      DIMENSION T(6),R(6),AK(6),LL3,LL4,LL5,LL6
                 B(6),RR(6),VE(6),VR(6),H(6),C(6),B9(6),D(6)
                 P(6),D(6),AB(6,6),U(6,6)
      READ(5,10) ...
      READ ...
      READ ...
      READ ...
      READ ...
      READ ...
      READ ...
      READ ...
      READ ...
   10 FORMAT( ... )
      DO 12 ...
   11 ...
   12 CONTINUE
      DO 20 ...
      B9X(I)=(R(I)*(-AK(I)*T(I))**(-1)*(EXP(-AK(I)*T(I))-EXP(-R(I)*T(I))))
      EXP(-R(I)*EXP(-R(I))*T(I)))/RR(I)
   20 PP ...
      VR(I)=EXP(-AK(I)*T(I)) ...
   30 VR(I)=162.0
      DO 40 I=1,6 VR(I-1)=VE(I-1)
   40 B(I)= VE(I) / (VE(I)+VR(I))
C CALCULATION OF REACH NUMBER ONE
      AO= ...
      DB0 ...
      D(1,1) ... E(1) PPO
      LL1 ... B(1) J0*PPO
      FE(1,1)=HH=DL1+C(1)*DB0+Q(1)*F(1)*K3(1)*P(1)
      AB(1,1) ... AND D(2,1)=D(1,1)
C CALCULATION OF U(2,2) AND D(2,1) FOR REACH NUMBER TWO
      D(2,2) ... B(2) ...
      L2 ...*LL2 ...+C(2)*FF1 + Q(2)*F(2)*K3(2)*P(2)
      AB(2,2) ...
      FF2 ... *D(2,2)
C CALCULATION OF U FOR REACH NUMBER 3
      D(3,1) ...
      LL3 ...*LL2 ...+LL3 ... C(3)*AB(2,1)
      AB(3,3) ... C(3)*FF2 + Q(3)*F(3)*K3(3) *P(3)
      FF3 ... *D(3,3)
C CALCULATION OF U FOR REACH NUMBER 4
      D(4,1) ...
      LL4 ... C(4)*AB(3,1)
      AB(4,4) ... C(4)*FF3 + Q(4)*F(4)*K3(4)*P(4)
      FF4 ... *D(4,4)
C CALCULATION OF U(5) FOR REACH NUMBER 5
      D(5,1) ...
```

```
      D(5,5) = A(5)*E(5)
      AB(5,1) = BB(4)*A(4)
      AB(5,2) = CC(5)*AB(4,1)
      AB(5,3) = DD(5)*AB(4,2)
      AB(5,4) = EE(5)*AB(4,3)
      AB(5,5) = HH(5)*AB(4,4)
      FF5 = FF5 + Q(5)*F(5)*KS(5)*P(5)
      U(5,5) = ABS(AB(5,5)/FF5)
C     CALCULATION OF ABS... FOR REACH NUMBER SIX
      D(6,6) = A(6)*E(6)
      AB(6,1) = BB(5)*A(5)
      AB(6,2) = CC(6)*AB(5,1)
      AB(6,3) = DD(6)*AB(5,2)
      AB(6,4) = EE(6)*AB(5,3)
      AB(6,5) = HH(6)*AB(5,4)
      AB(6,6) = HH(6)*AB(5,5)
      FF6 = FF6 + Q(6)*F(6)*KS(6)*P(6)
      U(6,6) = ABS(AB(6,6)/FF6)

   70 FORMAT(1H1,60X,'LIST OF INPUT DATA',/59X,' ==================')
   80 FORMAT(6/32X,7X,'REACH-1',7X,'REACH-2',7X,'REACH-3',7X,'REACH-4',7X,
     *'REACH-5',7X,'REACH-6')
  100 WRITE(6,90)(T(I),I=1,6)
   90 FORMAT(1H+,20X,'T(I)',/)
  110 WRITE(6,110)(R(I),I=1,6)
      FORMAT(1H+,20X,'R(I)',/)
  190 WRITE(6,190)(AK(I),I=1,6)
      FORMAT(1H+,20X,'AK(I)',/)
  120 WRITE(6,120)(VE(I),I=1,6)
      FORMAT(1H+,20X,'VE(I)',/)
  130 WRITE(6,130)(E(I),I=1,6)
      FORMAT(1H+,20X,'E(I)',/)
  140 WRITE(6,140)(F(I),I=1,6)
      FORMAT(1H+,20X,'F(I)',/)
  150 WRITE(6,150)(RR(I),I=1,6)
      FORMAT(1H+,20X,'RR(I)',/)
  160 WRITE(6,160)(Q(I),I=1,6)
      FORMAT(1H+,20X,'Q(I)',/)
  170 WRITE(6,170)(S(I),I=1,6)
      FORMAT(1H+,20X,'S(I)',/)
  188 WRITE(6,160)(KS(I),I=1,6)
   91 FORMAT(1H+,20X,'KS(I)',/)
      FORMAT(25X,6(7X,F7.4))
      FORMAT(25X,6(6X,F6.4))
   60 WRITE(6,60)(U(I,J),'CALCULATED U(I,J)',/59X,' ==================')
   30 FORMAT(1H0,/30X,(U(7,J),6(F7.2,7X)))
      CONTINUE
      STOP
      END
```

Appendix 3

INPUT DATA AND CALCULATED u_{ij}

FOR PROBLEMS P_1 AND P_2

LIST OF INPUT DATA
****

	REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
T(I)	4.0000	0.5200	0.7000	2.6200	0.3700	0.9500
R(I)	0.0000	0.3300	0.2240	0.2160	0.2160	0.2500
AK(I)	0.1260	0.1260	0.1260	0.1260	0.1260	0.1260
VE(I)	16.0000	32.5000	6.4000	41.5000	48.5000	20.0000
E(I)	100.0000	436.0000	179.0000	36.0000	4.8300	661.0000
F(I)	0.7000	0.7000	0.7000	0.7000	0.7000	0.7000
RR(I)	4.0000	2.4400	3.4100	3.3800	3.2000	2.9000
Q(I)	3.2800	3.2800	3.2800	3.2800	3.2800	3.2800
S(I)	2.1800	2.1800	2.1800	2.1800	2.1800	2.1800
KS(I)	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000

V_{R_0} 1162

L_{b_0} 1.0

D_{b_0} 1.0

CALCULATED U(I,J)
****

REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
0.7756	0.0000	0.0000	0.0000	0.0000	0.0000
0.8504	0.9172	0.0000	0.0000	0.0000	0.0000
0.8604	2.0082	0.1029	0.0000	0.0000	0.0000
0.8540	4.3310	0.3305	0.4018	0.0000	0.0000
0.0433	4.0959	0.3476	0.4337	0.0120	0.0000
0.0162	4.0280	0.3827	0.5003	0.0379	1.6400

Appendix 4

INPUT DATA AND CALCULATED u_{ij}
FOR PROBLEM P_3

LIST OF INPUT DATA
****

	REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
T(I)	4.0000	0.5200	0.7000	2.6200	0.3700	0.9500
B(I)	0.0800	0.3300	0.2240	0.2160	0.2160	0.2500
AK(I)	0.1260	0.1260	0.1260	0.1260	0.1260	0.1260
VE(I)	16.0000	32.5000	6.4000	41.5000	48.5000	20.0000
E(I)	148.0000	436.0000	179.0000	38.0000	4.8300	661.0000
F(I)	0.7000	0.7000	0.7000	0.7000	0.7000	0.7000
HR(I)	4.8800	2.4400	3.4100	3.3800	3.2000	2.9000
Q(I)	3.2800	3.2800	3.2800	3.2800	3.2800	3.2800
S(I)	1.3471	1.5027	1.5727	1.7157	1.7283	1.8212
KS(I)	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000

V_{R_0} = 1162

L_{b_0} = 1.0

D_{b_0} = 1.0

CALCULATED U(I,J)
****

	REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
	20.0711	0.0	0.0	0.0	0.0	0.0
	9.6622	10.3734	0.0	0.0	0.0	0.0
	5.8928	13.4407	0.6288	0.0	0.0	0.0
	2.8878	14.6457	1.1178	1.3588	0.0	0.0
	2.7505	14.6642	1.1338	1.4147	0.0393	0.0
	1.9004	11.7645	0.9323	1.2189	0.0923	3.9955

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